UNIVERSITY OF LIVERPOOL

21st String Phenomenology – 5th July 2022

Gravity at the Tip of the Throat

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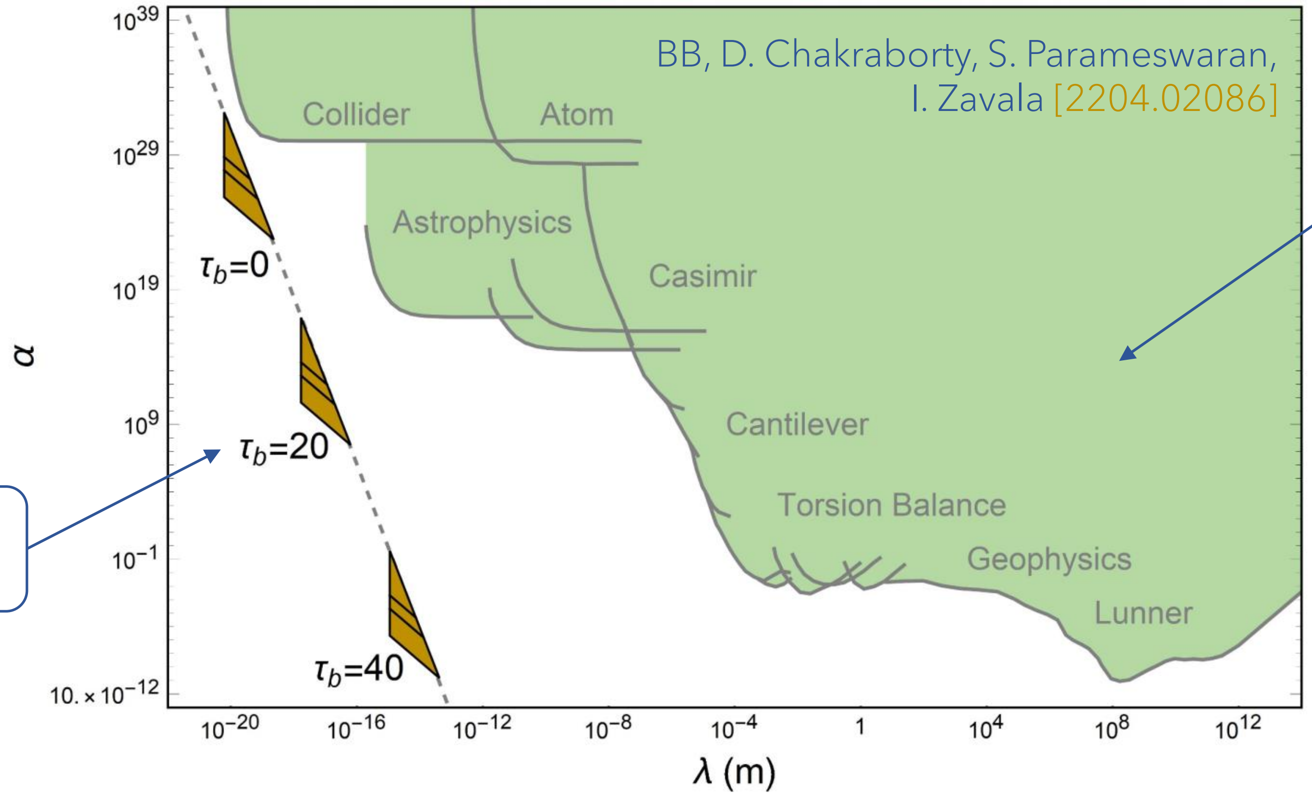
Based on [2204.02086] in collaboration with:

Dibya Chakraborty, Susha Parameswaran, Ivonne Zavala

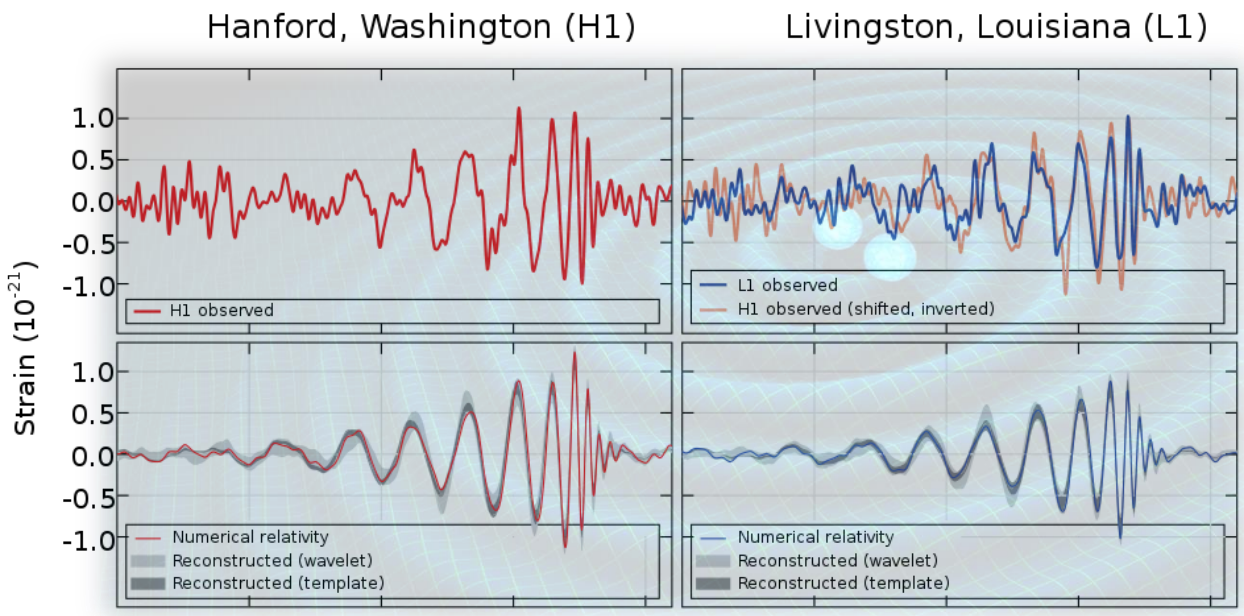
Spoiler

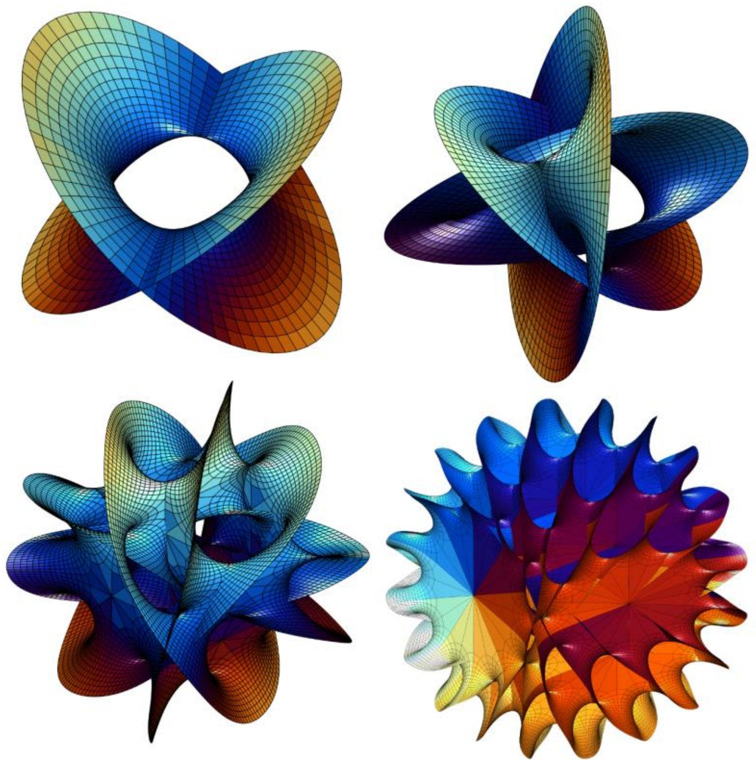


String Theory

Experiment

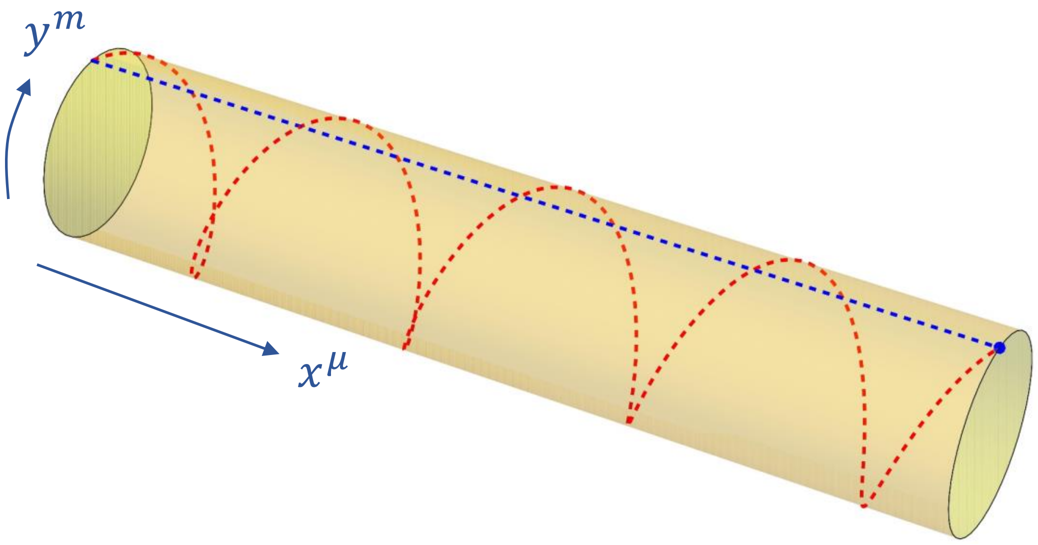
The Question





Can GWs tell us about (warped) extra dimensions?

Kaluza-Klein modes



m_4

m_3

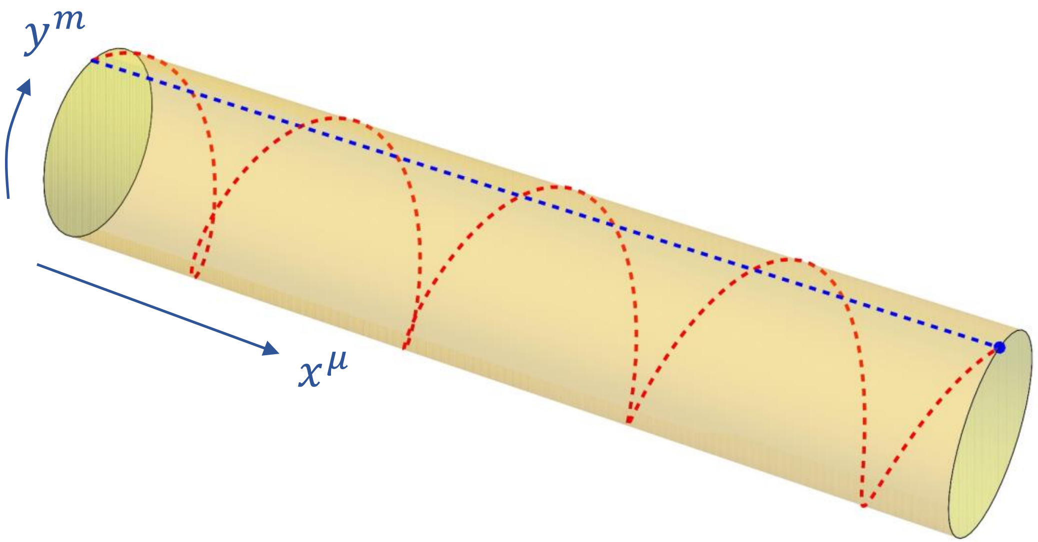
$$p^\mu p_\mu = \underbrace{-m^2 - p^n p_n}_{m_k^2}$$

m_2

M_{KK}

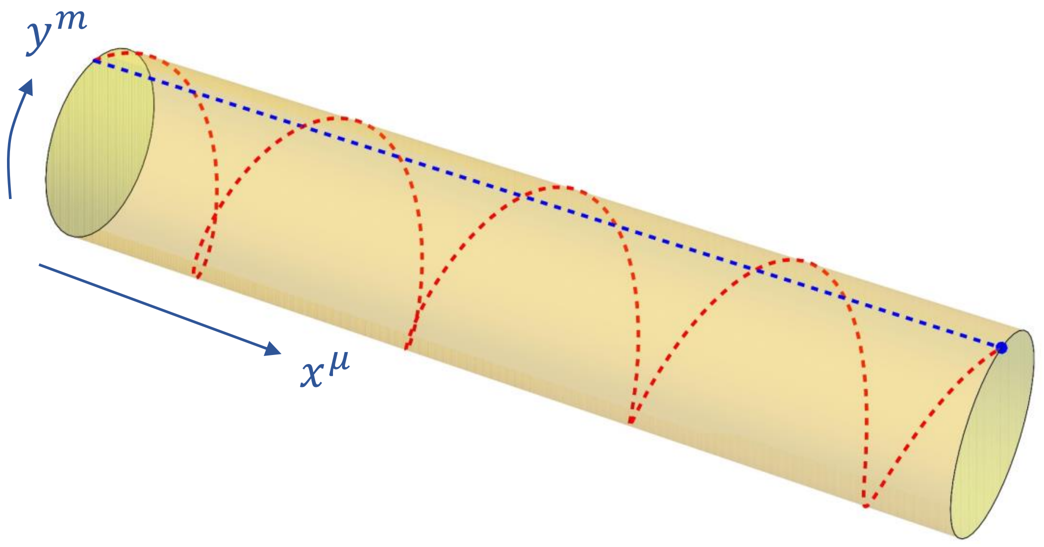
The tower is the signature of extra dimensions.

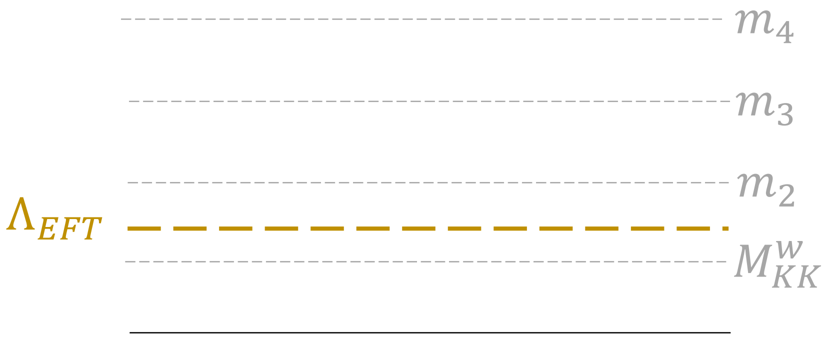
Kaluza-Klein modes



The tower is the signature of extra dimensions.

Kaluza-Klein modes





The tower is the signature of extra dimensions.

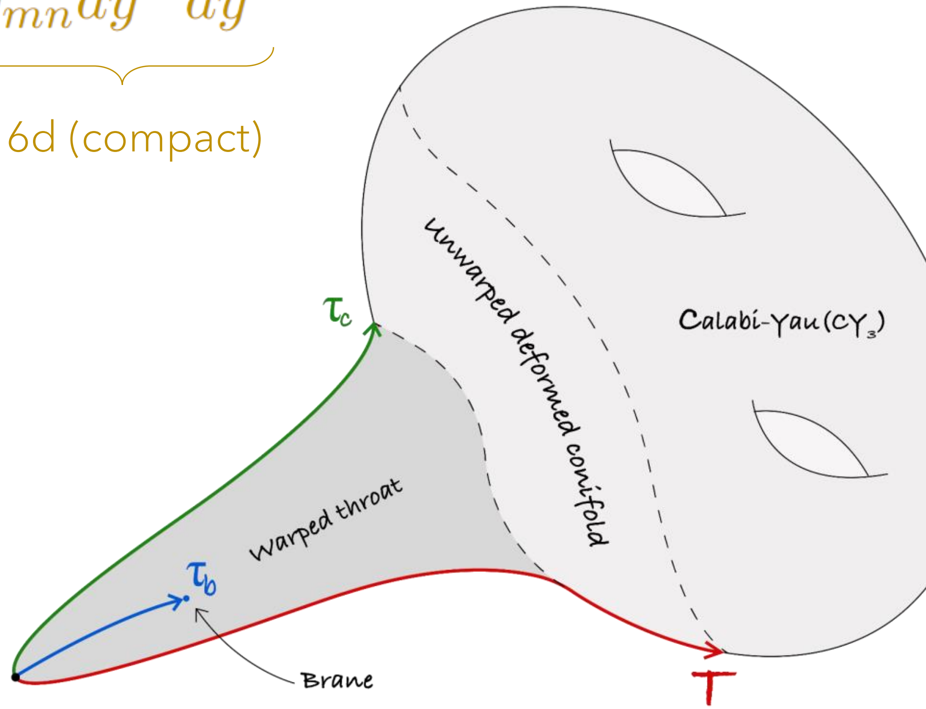
Warped compactification

String Theory: 10d → 4d

$$ds^2 = \underbrace{H(y)}_{\text{Warp factor}}{}^{-1/2}\underbrace{g_{\mu\nu}dx^\mu dx^\nu}_{\text{4d}} + H(y)^{1/2}\underbrace{\tilde{g}_{mn}dy^m dy^n}_{\text{6d (compact)}}$$

The warping makes scales position (y^m) dependent

What does it do to Gravitational Waves?



KK Gravitational Waves

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G}\,(R + \mathcal{L}_{\mathrm{IIB}}) + S_{\mathrm{brane}}$$

10d gravity (e.g. Type IIB)

$$\downarrow$$

$$\Box_{10} h_{MN} + 2\bar{R}_{MPNQ} h^{PQ} = T^{(1)}_{MN}$$

10d GWs

$$\downarrow$$

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

4d GWs (tower)

KK Gravitational Waves

(e.g. Type IIB)

$$h_{\mu\nu}(x,y) = \sum_k \boxed{h^k_{\mu\nu}(x)}\,\boxed{\phi_k(y)}$$

wavefunctions

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

d GWs

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

4d GWs (tower)

$$h_{MN} \to \boxed{h_{\mu\nu}},\ h_{\mu n},\ h_{mn}$$

KK Gravitational Waves

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu} \quad \leftrightarrow \quad \text{Tower of GWs } (f_{GW} \sim m_k)$$

OBSERVATION

f_{GW} cannot* be too high (10^4 Hz ~ 10^{-23} eV)

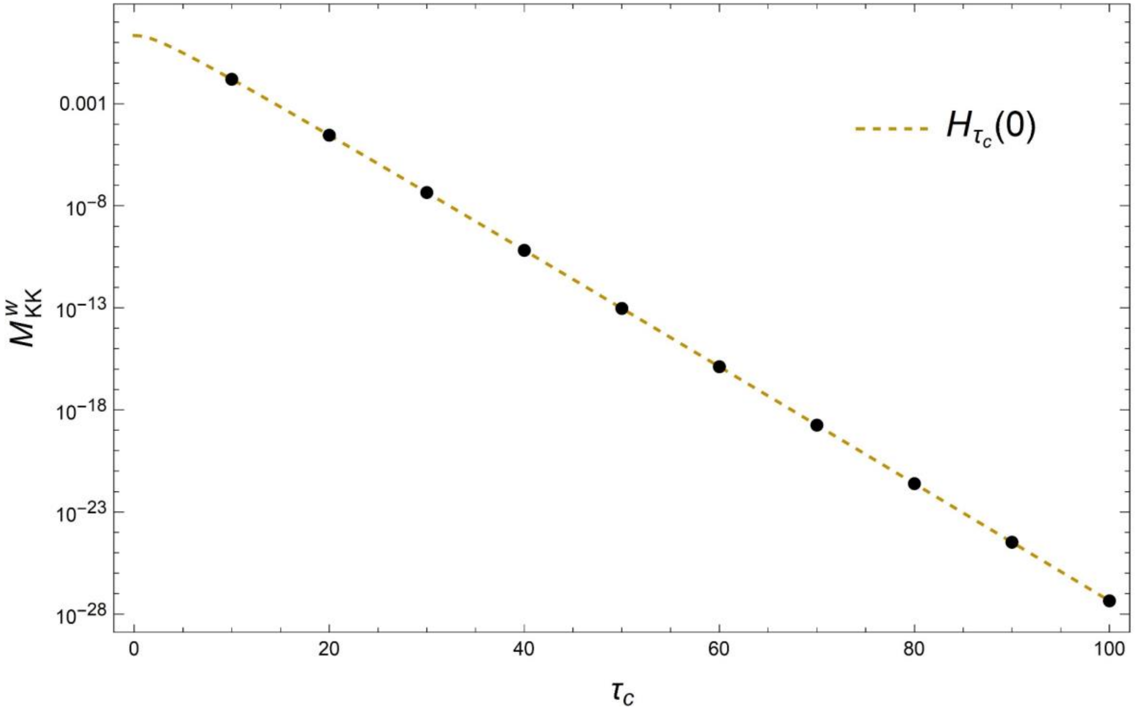
QUESTION

Wouldn't we have seen $h^k_{\mu\nu}$ already?

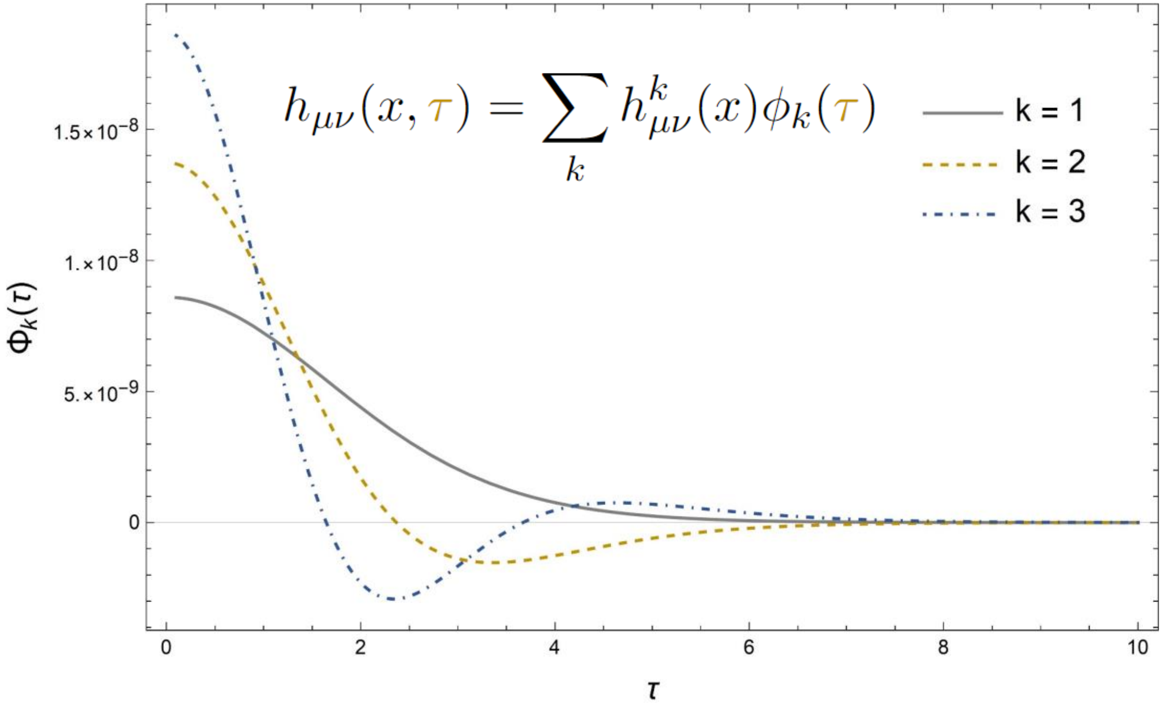
H. Firouzjahi and S. H. H. Tye [hep-th/0512076]
G. Shiu, B. Underwood, K. M. Zurek, and D. G. E. Walker [0705.4097]

KK Gravitons

*Fully warped limit (τ_c = T)



Masses



Wavefunctions

Braneworld

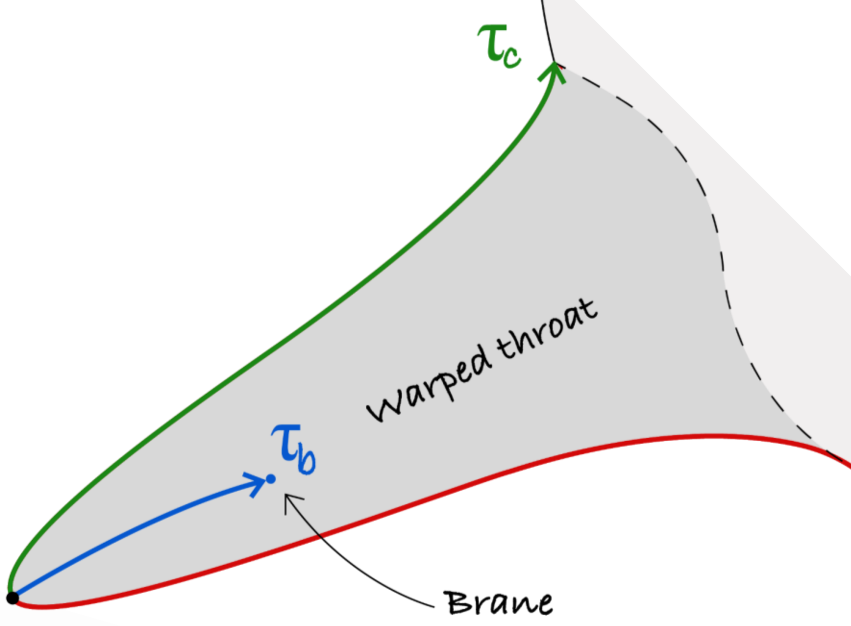
For a brane somewhere in the throat

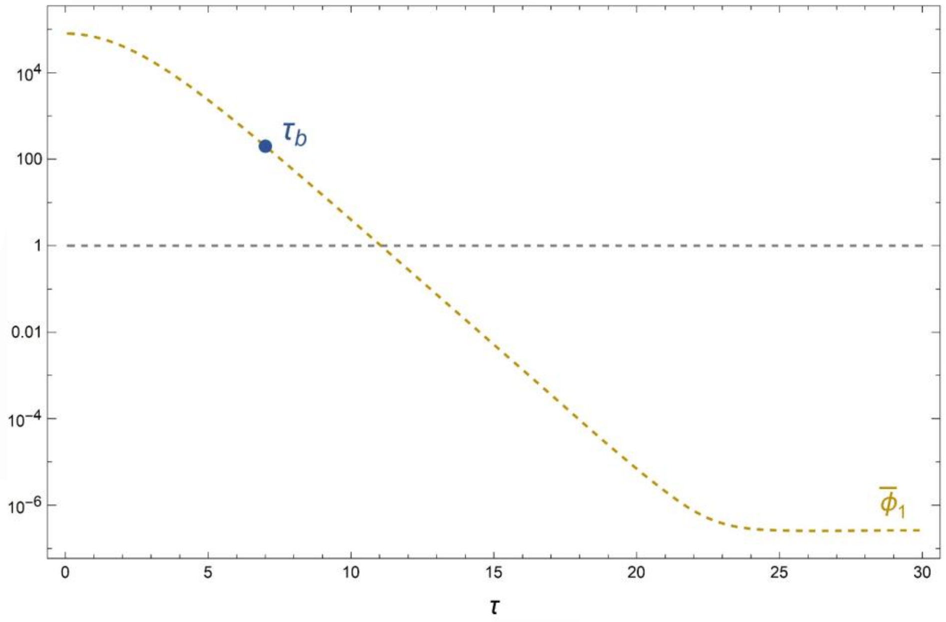
$$\mathcal{L}_{\text{brane}} \sim \sum_k \boxed{\frac{\bar{\phi}_k(\tau_b)}{M_P}} h^k_{\mu\nu} T^{\mu\nu}$$

Couplings

Gravitational interactions on the brane include the whole tower

$$h_{\mu\nu}(x,y) = \sum_k h^k_{\mu\nu}(x)\phi_k(y)$$

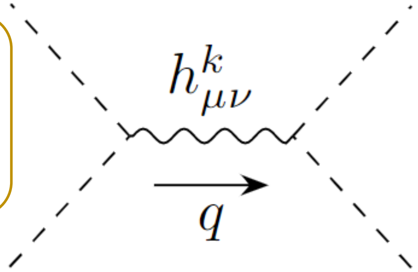




Do we see the extra dimensions?

The wavefunctions weigh the contribution of each mode

$$V(q) = \lim_{q^0 \to 0} \sum_k \boxed{\left| \frac{\bar{\phi}_k(\tau_b)}{M_P} \right|^2}$$



To compare with experiments we express it as

$$V(r) = G\frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Strength Range

Do we see the extra dimensions?

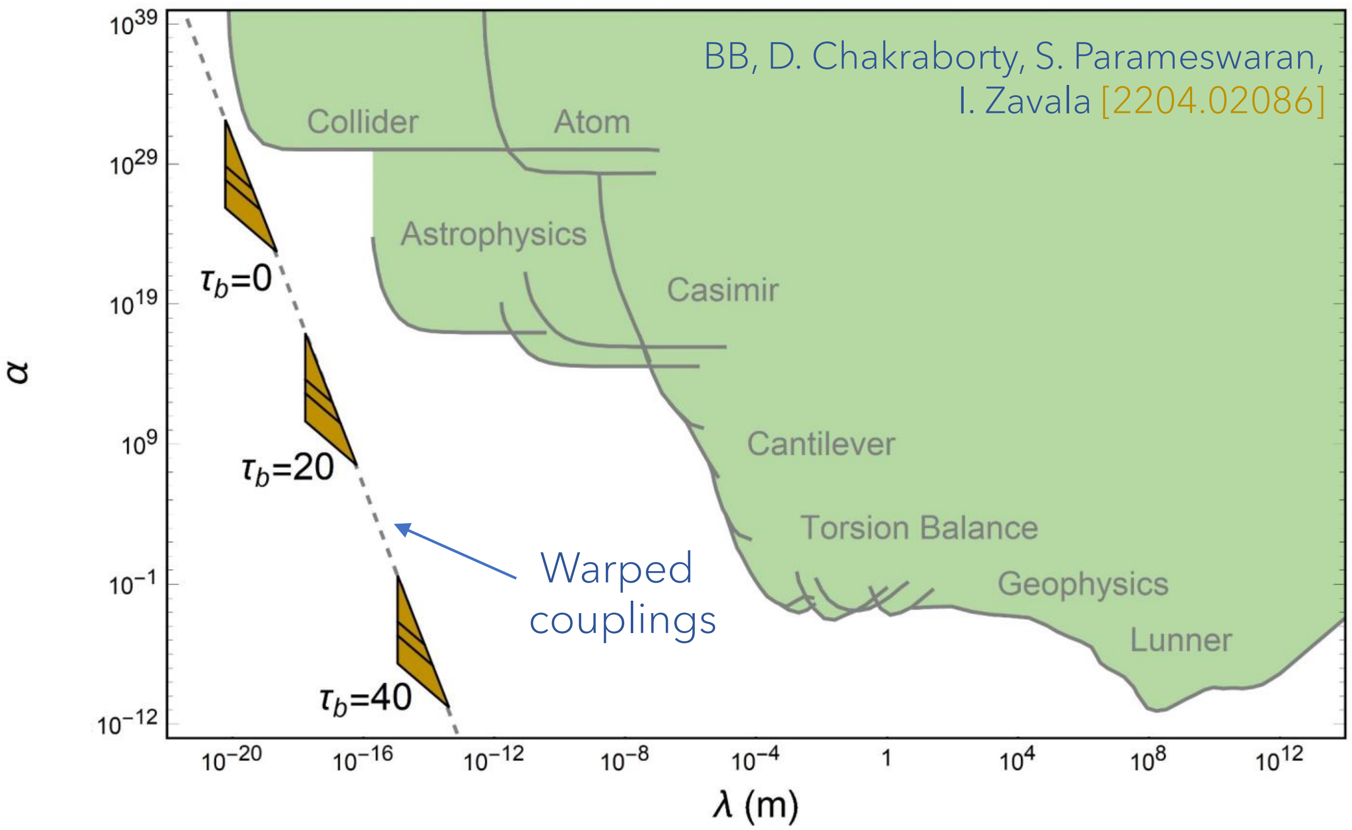
Predictions vs experimental constraints

$$|\bar{\phi}_k|^2 \sim \alpha \approx \frac{(2\pi)^2}{(g_s M)^3} \frac{2|\phi_1(\tau_b)|^2}{I(\tau_b)^{1/2}} \frac{g_s^2}{\mathcal{H}^2}$$

$$m_k \sim \lambda^{-1} \approx \frac{\mathcal{H}}{2^{1/6}} \frac{2\pi}{\sqrt{g_s M}} I(\tau_b)^{1/4}$$

Triangle regions:

$g_s M > 1$	Supergravity (α')
$g_s < 1$	String loop expansion
$M < M_{max}$	D3 Tadpole



Observational constraints from J. Murata, S. Tanaka [1408.3588]
J. A. R. Cembranos, A. L. Maroto, and H. Villarrubia-Rojo [1706.07818]

Conclusions

- (compact) Extra dimensions $\longrightarrow$ Tower of 4d gravitons $\longrightarrow$ Tower of 4d GWs
- Phenomenological parameters $(\alpha, \lambda) \leftrightarrow$ String theory parameters $(g_s, M, \dots)$

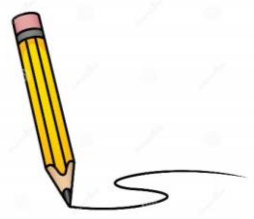
  No conflict

- Wavefunction profiles are important! What's the effect on GWs?

Thank You

BACKUP SLIDES

KK Gravitational Waves



Higher dimensional GWs

$$\Box_{10} h_{MN} - 2\bar{R}^{S}{}_{MNP} g^{PQ} h_{QS} = T^{(1)}_{MN}$$

give rise 4d GWs

$$\Box_4 h_{\mu\nu} + \underbrace{\left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right)}_{\text{6d (compact)}} h_{\mu\nu} = T^{(1)}_{\mu\nu}$$

$$h_{\mu\nu}(x, y) = \sum_k h^k_{\mu\nu}(x)\phi^k(y)$$

KK Gravitational Waves

Higher dimensional GWs

$$\Box_{10} h_{MN} - 2 \ldots$$

give rise 4d GWs

$$\Box_4 h_{\mu\nu} + \Big(\ldots$$

Define the functions as

$$\left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right)\phi^k(y) = -m_k^2\phi^k(y)$$

$\Rightarrow$ Infinitely many orthogonal modes.

6d (compact)

$$h_{\mu\nu}(x,y) = \sum_k h^k_{\mu\nu}(x)\phi^k(y)$$

KK Gravitational Waves

The decomposition is defined by

$$\left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right)\phi^k(y) = -m_k^2\phi^k(y)$$

i.e. the functions $\phi^k(y)$ are eigenfunctions with eigenvalue $-m_k^2$ and orthogonal

$$\int d^6y\sqrt{g_6}\; H(y)\phi_k(y)\phi_{k'}(y) = \mathcal{N}_{(k)}^2\delta_{kk'}$$

Infinitely many solutions (discrete) $\Rightarrow$ Tower of states

KK Gravitational Waves

$$\Box_4 h_{\mu\nu} + \left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right)h_{\mu\nu} = 2\tilde{T}^{(1)}_{\mu\nu}$$

$$h_{\mu\nu}(x,y) = \sum_{k'} h^{k'}_{\mu\nu}(x)\phi_{k'}(y)$$

$$\sum_{k'}\left(\Box_4 h^{k'}_{\mu\nu} - m^2_{k'}h^{k'}_{\mu\nu}\right)\phi_{k'}(y) = 2\tilde{T}^{(1)}_{\mu\nu}(x,y)$$

$$\int d^6y\sqrt{g_6}\ H(y)\phi_k(y)$$

$$\boxed{\Box_4 h^k_{\mu\nu} - m^2_k h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}}$$

$$\mathrm{T}_{\mu\nu} \equiv 2\int d^6y\sqrt{g_6}\ H(y)\phi_k(y)\tilde{T}^{(1)}_{\mu\nu}(x,y)$$

KK Gravitational Waves

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu} \qquad \mathrm{T}_{\mu\nu} \equiv 2 \int d^6 y \sqrt{g_6}\; H(y)\phi_k(y)\tilde{T}^{(1)}_{\mu\nu}(x,y)$$

The warped extra dimensions contribute with

→ Tower of GWs ($k > 0$) (high frequencies $f_{GW} \sim m_k$)

→ More sources

→ Bulk (e.g. h_{mn})

→ Localised (e.g. brane fields)

→ Warped energy-momentum

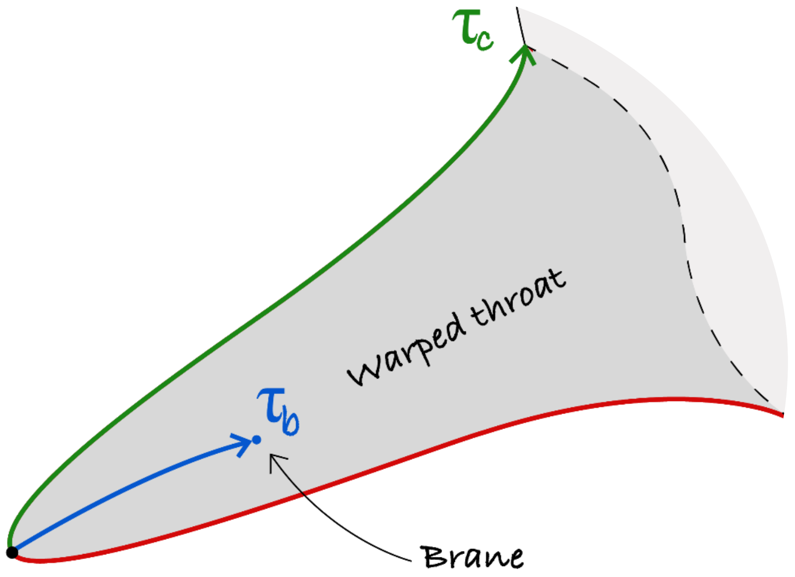
Warped deformed conifold

Klebanov-Strassler (KS) solution

$$ds^2 = H(y)^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + H(y)^{1/2} c^{1/2} g_{mn} dy^m dy^n$$

$$g_{mn} = \begin{cases} g_{mn}^{(\mathrm{KS})} & H(y) \gg 1 \\ g_{mn}^{(\mathrm{CY})} & H(y) \sim 1 \end{cases}$$

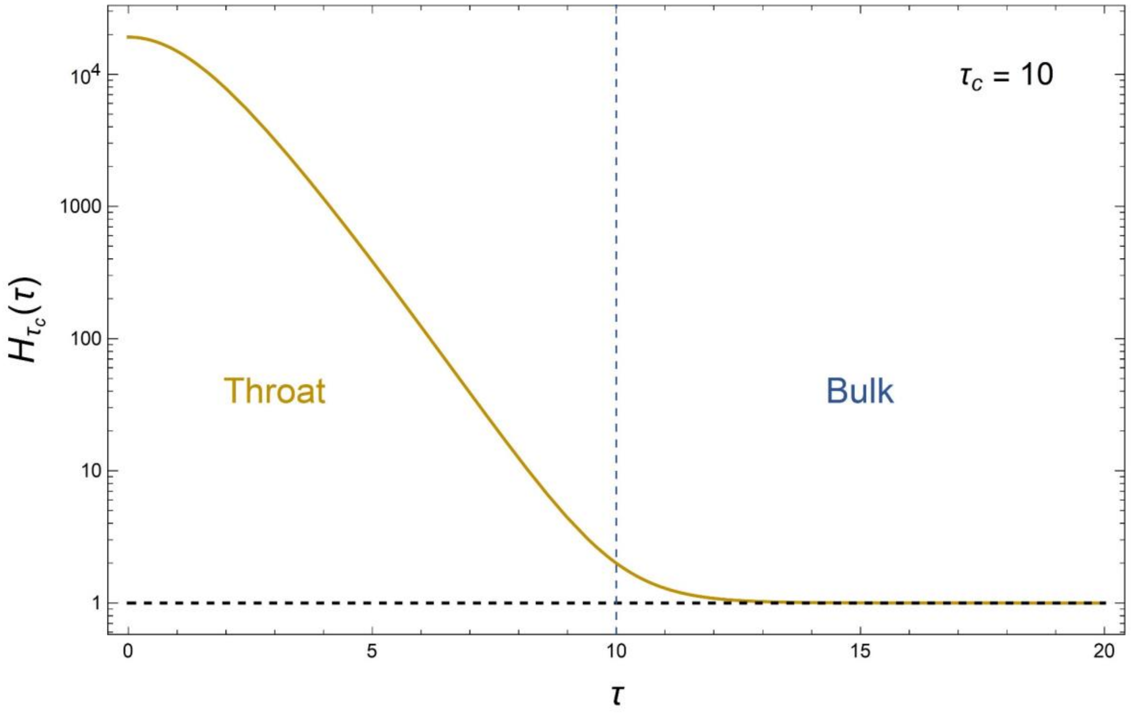
$$g_{mn}^{(KS)} = \frac{\epsilon^{4/3}}{2} \mathcal{K}(\tau) \begin{pmatrix} \frac{1}{3\mathcal{K}^3(\tau)} \mathbb{1}_2 & 0 & 0 \\ 0 & \sinh^2(\tau/2) \mathbb{1}_2 & 0 \\ 0 & 0 & \cosh^2(\tau/2) \mathbb{1}_2 \end{pmatrix}$$

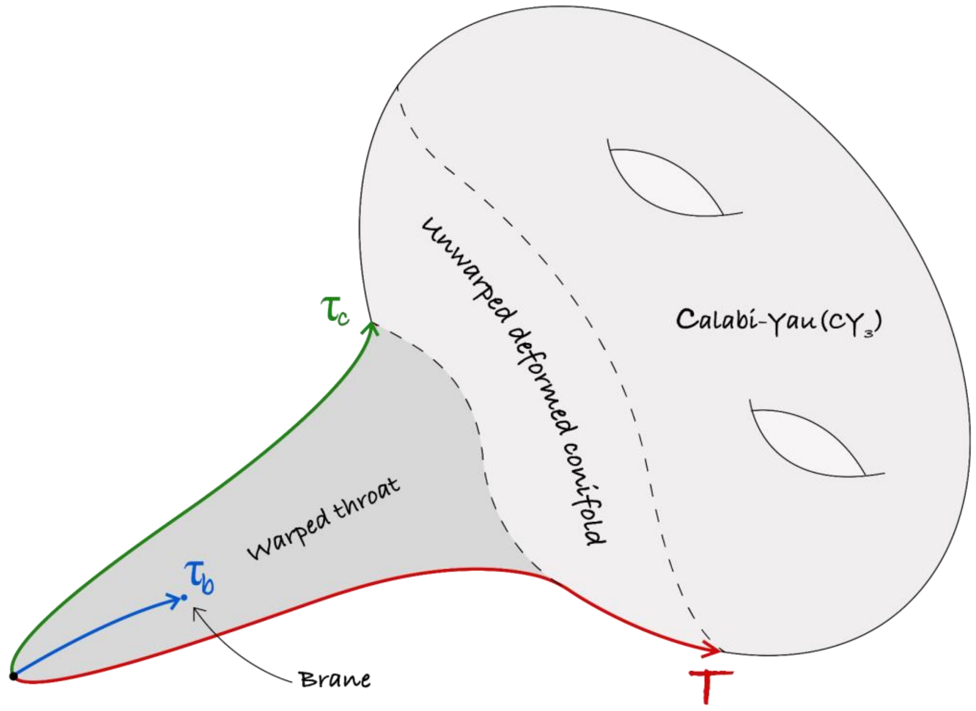


Warped compactification

KS + Bulk

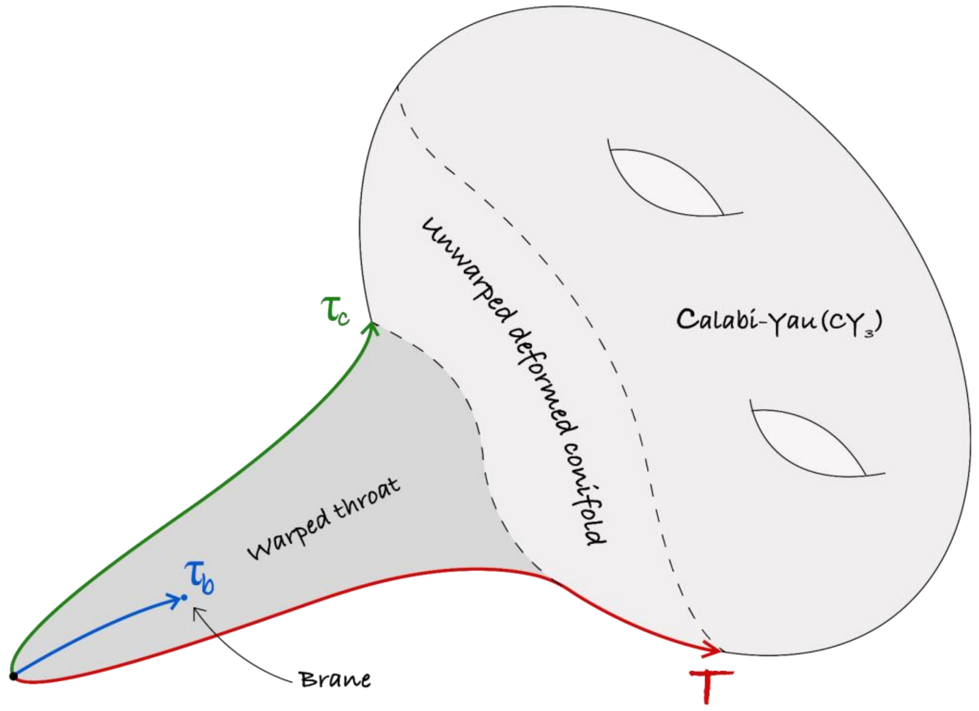
$$ds^2 = H(y)^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + H(y)^{1/2} c^{1/2} g_{mn} dy^m dy^n$$





Our paper [2204.02086]

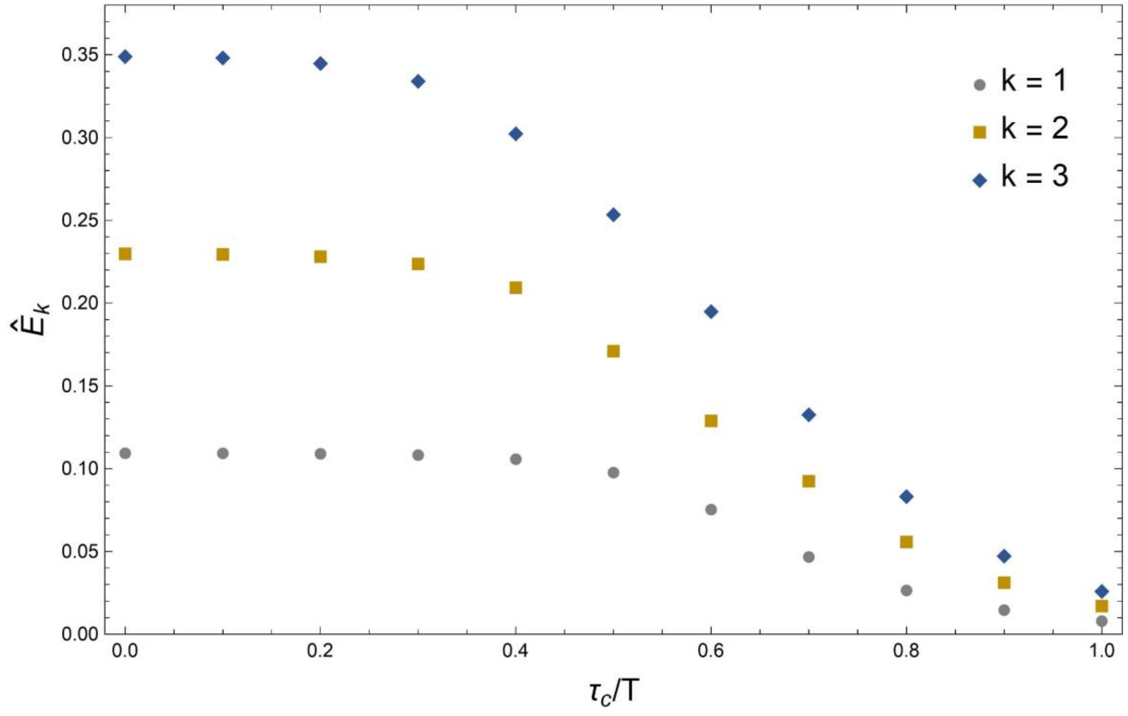
1. 4d Minkowski
2. Trivial angular solutions ($\phi_k = \phi_k(\tau)$)
3. Unwarped conifold region ($\tau_c < \tau <$ T)
4. Vanishing boundary conditions on CY3 ($\tau =$ T)
5. (3+1)-brane somewhere in the conifold ($\tau_b <$ T)



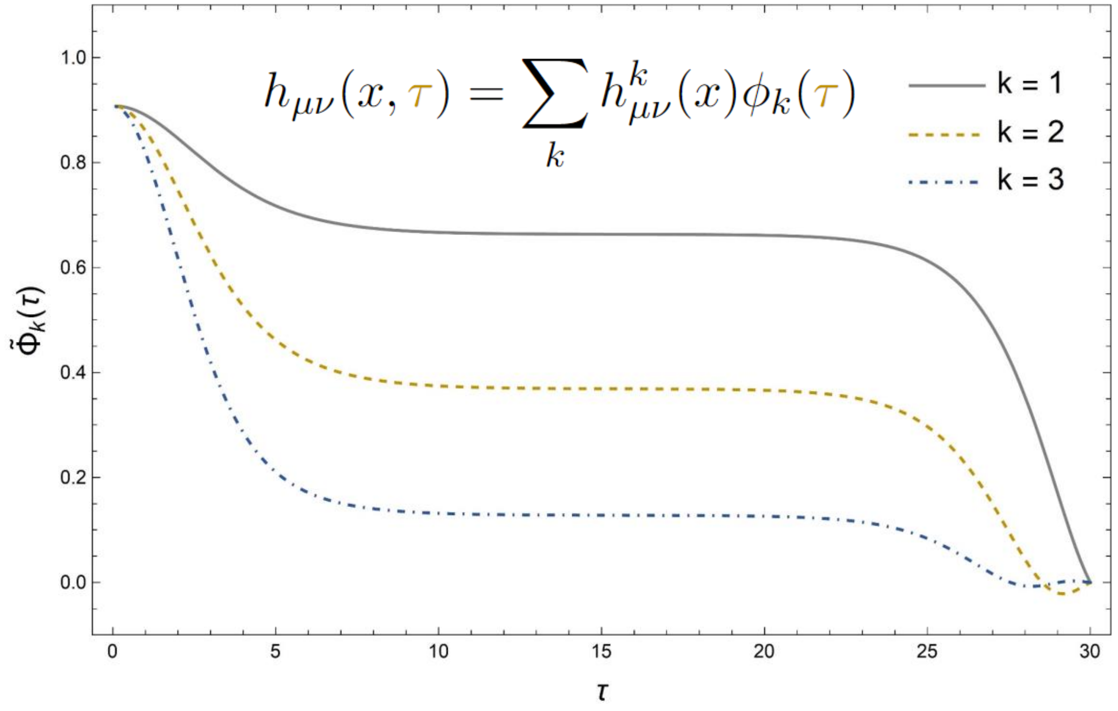
Throat vs Bulk

There is a competition between warping and bulk size.

cf. H. Firouzjahi and S. H. H. Tye [hep-th/0512076]



$\tau_c/T = 0.5$



Gravitational Waves

Each mode in the tower has its own wave equation

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

Tower of frequencies $\omega_k \sim m_k \quad (\Delta\omega = M^w_{KK})$

	f_{GW} (Hz)	M^w_{KK} (eV)	τ_b	τ_c	$z^{1/3}$	r_{UV}	$\mathcal{V}_{th}$	MK
LISA	10^0	10^{-27}	195	239	1.51×10^{-47}	1.70	290	3259
LIGO-Virgo/ET	10^4	10^{-23}	168	211	1.51×10^{-43}	1.64	240	2906
UHF	10^9	10^{-18}	133	176	1.51×10^{-38}	1.57	183	2464

cf. D. Andriot and G. Lucena Gómez [1704.07392]
cf. A. K. Mishra, A. Ghosh, S. Chakraborty [2106.05558]